

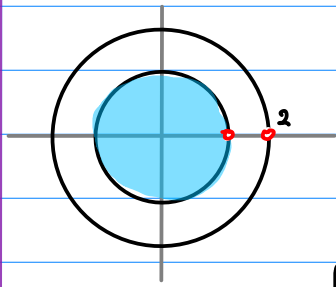
# Laurent Series and z-Transform Examples case 3.A

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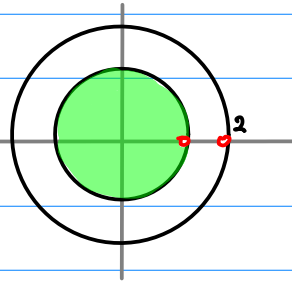
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I



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

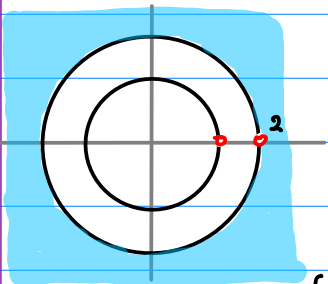
$$f(z) = \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

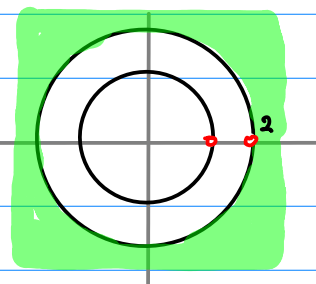
$$X(z) = \sum_{n=-\infty}^{\infty} [2^{n-1} - 1] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

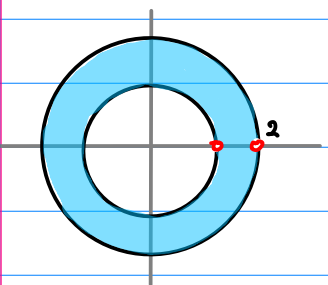
$$f(z) = \sum_{n=-1}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^n$$



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

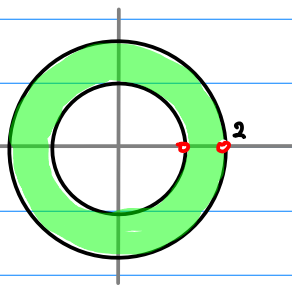
$$X(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^{-n}$$

III



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$

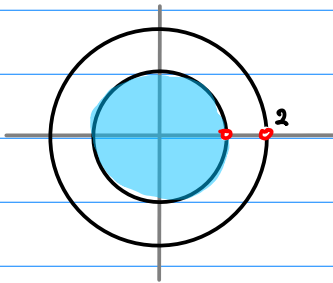


$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

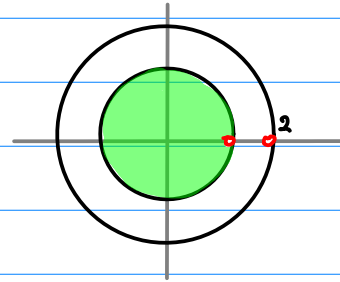
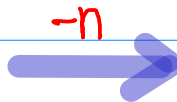
$$X(z) = \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

3. A

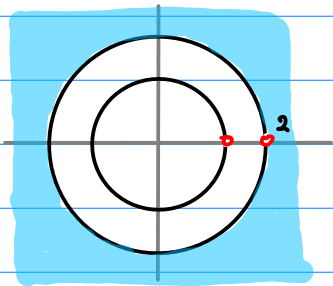
$$f(z) = \frac{-1}{(z-1)(z-2)} \equiv X(z) = \frac{-1}{(z-1)(z-2)}$$



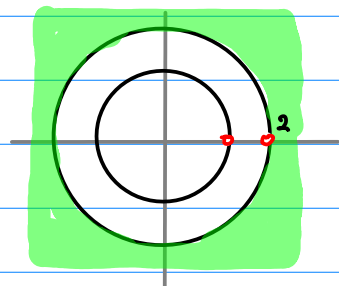
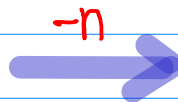
$$\sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$



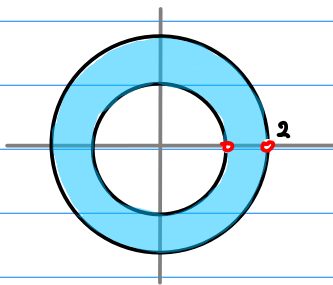
$$\sum_{n=0}^{\infty} \left[ 2^{n-1} - 1 \right] z^{-n}$$



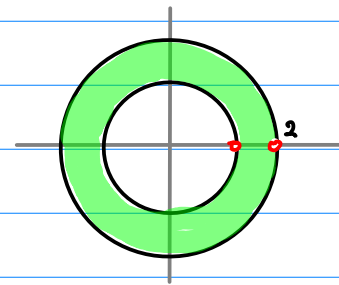
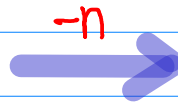
$$\sum_{n=-1}^{\infty} \left[ 1 - \left(\frac{1}{2}\right)^{n+1} \right] z^n$$



$$\sum_{n=-1}^{\infty} \left[ 1 - 2^{n-1} \right] z^{-n}$$

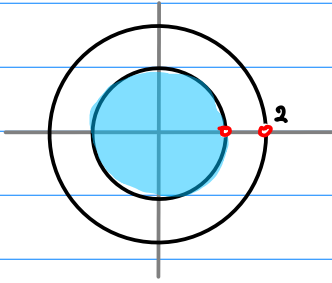


$$\sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

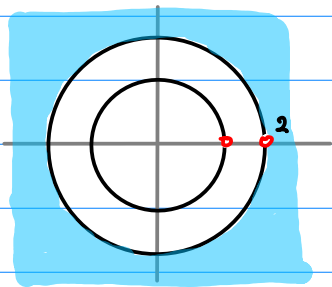


$$+ \sum_{n=-1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

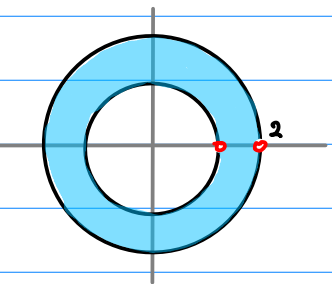
$$f(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{(z-1)} - \frac{1}{(z-2)}$$



$$\begin{aligned} & - \frac{(+)}{1 - (\frac{z}{1})} + \frac{(\frac{1}{2})}{1 - (\frac{z}{2})} \\ &= - \sum_{n=0}^{\infty} (1) (\frac{z}{1})^n + \sum_{n=0}^{\infty} (\frac{1}{2}) (\frac{z}{2})^n \\ &= - \sum_{n=0}^{\infty} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n \\ &= \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^n \end{aligned}$$

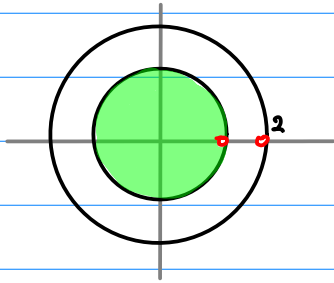


$$\begin{aligned} & + \frac{(\frac{1}{2})}{1 - (\frac{1}{z})} - \frac{(\frac{1}{2})}{1 - (\frac{z}{2})} \\ &= + \sum_{n=0}^{\infty} (\frac{1}{z}) (\frac{1}{z})^n - \sum_{n=0}^{\infty} (\frac{1}{2}) (\frac{z}{2})^n \\ &= \sum_{n=0}^{\infty} [1 - 2^n] z^{-n-1} \\ &= \sum_{n=-1}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^n \end{aligned}$$



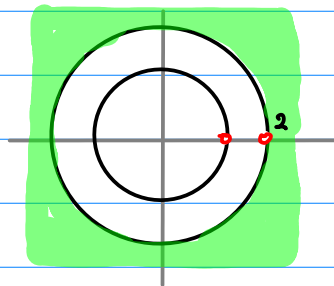
$$\begin{aligned} & + \frac{(\frac{1}{z})}{1 - (\frac{1}{z})} + \frac{(\frac{1}{2})}{1 - (\frac{z}{2})} \\ &= + \sum_{n=0}^{\infty} (\frac{1}{z}) (\frac{1}{z})^n + \sum_{n=0}^{\infty} (\frac{1}{2}) (\frac{z}{2})^n \\ &= + \sum_{n=0}^{\infty} (\frac{1}{z}) (\frac{1}{z})^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n \\ &= + \sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n \end{aligned}$$

$$X(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{(z-1)} - \frac{1}{(z-2)}$$



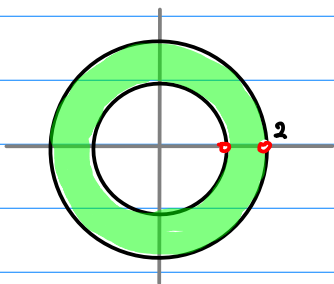
$$\sum_{n=0}^{\infty} [2^{n-1} - 1] z^{-n}$$

$$\begin{aligned} & -\frac{(+)}{1 - (\frac{z}{1})} + \frac{(\frac{1}{2})}{1 - (\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty} (1) \left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{z}{2}\right)^n \\ &= -\sum_{n=0}^{\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n \\ &= \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n \end{aligned}$$



$$\sum_{n=-1}^{\infty} [1 - 2^{n-1}] z^{-n}$$

$$\begin{aligned} & +\frac{(\frac{1}{2})}{1 - (\frac{1}{z})} - \frac{(\frac{1}{2})}{1 - (\frac{z}{2})} \\ &= +\sum_{n=0}^{\infty} \left(\frac{1}{z}\right) \left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty} \left(\frac{1}{z}\right) \left(\frac{z}{2}\right)^n \\ &= \sum_{n=0}^{\infty} [1 - 2^n] z^{-n-1} \\ &= \sum_{n=-1}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^{-n} \end{aligned}$$



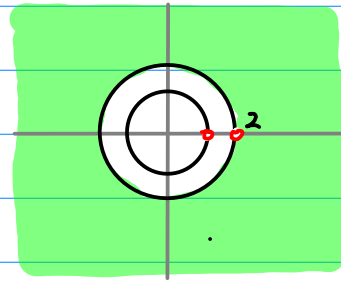
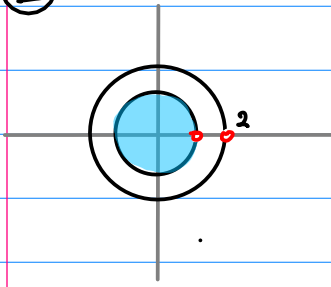
$$+\sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

$$\begin{aligned} & +\frac{(\frac{1}{z})}{1 - (\frac{1}{z})} + \frac{(\frac{1}{2})}{1 - (\frac{z}{2})} \\ &= +\sum_{n=0}^{\infty} \left(\frac{1}{z}\right) \left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{z}{2}\right)^n \\ &= +\sum_{n=0}^{\infty} \left(\frac{1}{z}\right) \left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n \\ &= +\sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n \end{aligned}$$



$$f(z) = \frac{-1}{(z-1)(z-2)} = X(z)$$

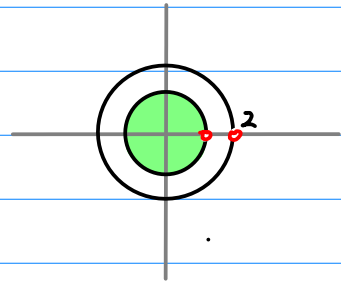
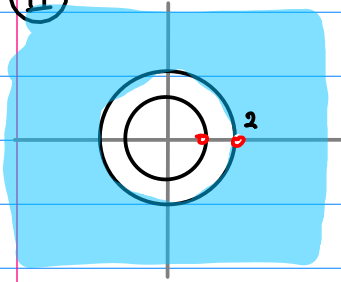
Ⓘ



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

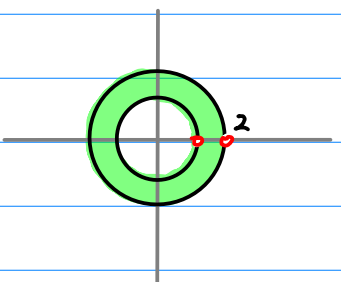
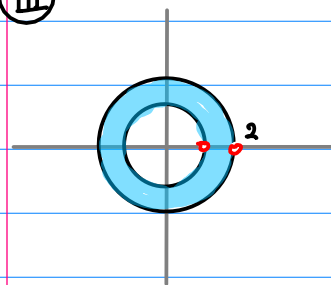
Ⓢ



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n+1} - 1 & (n \leq 0) \end{cases}$$

Ⓣ



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n+1} & (n \leq 0) \end{cases}$$

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{-n+1} - 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

$$X(z) = \sum_{n=0}^{-\infty} \left(\frac{1}{2}\right)^{-n+1} z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

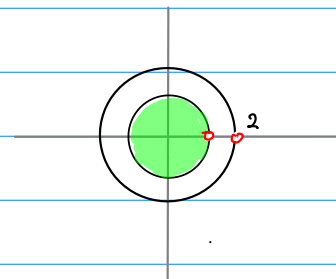
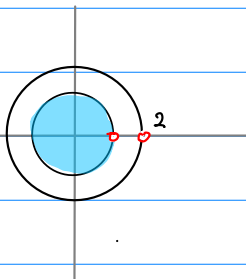
$$= \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{z}{2}\right)} - \frac{1}{1 - z}$$

$$= \frac{-1}{z-2} + \frac{1}{z-1}$$

$$= \frac{-z+1+z-2}{(z-1)(z-2)}$$

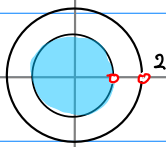
$$= \frac{-1}{(z-1)(z-2)}$$

$$\left|\frac{z}{2}\right| < 1 \quad \left|\frac{z}{1}\right| < 1$$



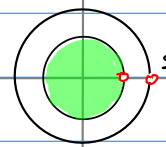


I



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

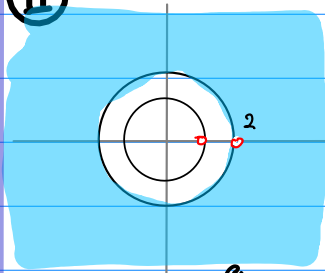
$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

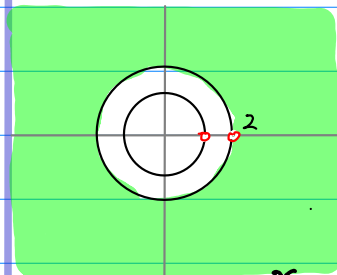
$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n > 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

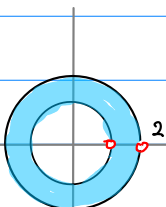
$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=-1}^{-\infty} 2^{-n-1} \cdot z^n$$



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

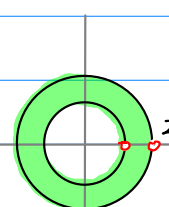
$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} \cdot z^{-n}$$

III



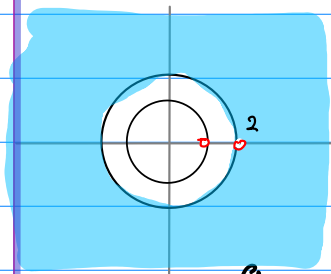
$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n$$



$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

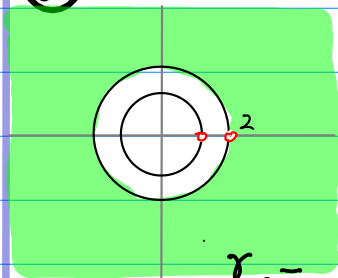
$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n}$$



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

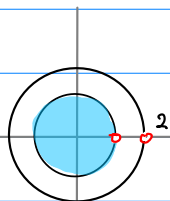
$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=-1}^{-\infty} 2^{-n-1} z^n$$

Ⓘ



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

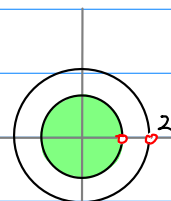
$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} z^{-n}$$



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

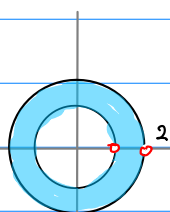
$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

Ⓛ



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n+1} - 1 & (n \leq 0) \end{cases}$$

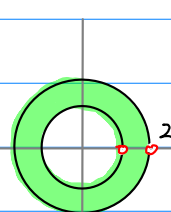
$$X(z) = \sum_{n=0}^{-\infty} 2^{n+1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n + \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n$$

Ⓜ



$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n+1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n+1} \cdot z^{-n} + \sum_{n=1}^{\infty} 1 \cdot z^{-n}$$



